Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Advanced/Advanced Subsidiary

Thursday 6 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43997A

1. A particle P of mass 2 kg is moving with velocity $(\mathbf{i} - 4\mathbf{j})$ m s^{-1} when it receives an impulse of $(3\mathbf{i} + 6\mathbf{j})$ N s.

Find the speed of P immediately after the impulse is applied.

(5)

---

2. A particle P of mass 3 kg moves from point A to point B up a line of greatest slope of a fixed rough plane. The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4.

Given that AB = 15 m and that the speed of P at A is 20 m s^{-1}, find

(*a*) the work done against friction as P moves from A to B,

(3)

(*b*) the speed of P at B.

(4)

---

3. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(*a*) the times when P is instantaneously at rest,

(3)

(*b*) the greatest speed of P in the interval $0 \leq t \leq 4$,

(5)

(*c*) the total distance travelled by P in the interval $0 \leq t \leq 4$.

(5)

4.

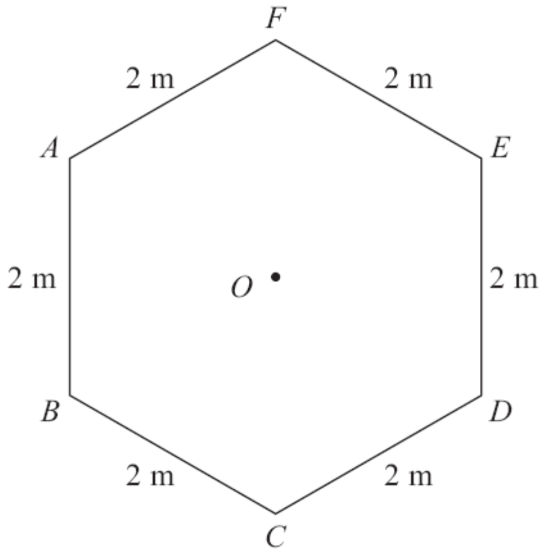


Figure 1

The uniform lamina *ABCDEF* is a regular hexagon with centre *O* and sides of length 2 m, as shown in Figure 1.

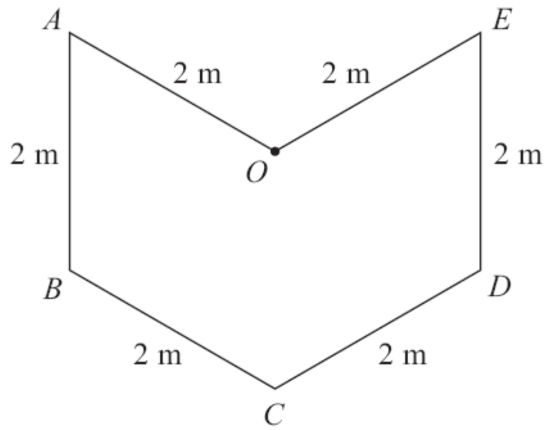


Figure 2

The triangles *OAF* and *OEF* are removed to form the uniform lamina *OABCDE*, shown in Figure 2.

(*a*) Find the distance of the centre of mass of *OABCDE* from *O*.

(5)

The lamina *OABCDE* is freely suspended from *E* and hangs in equilibrium.

(*b*) Find the size of the angle between *EO* and the downward vertical.

(6)

5.

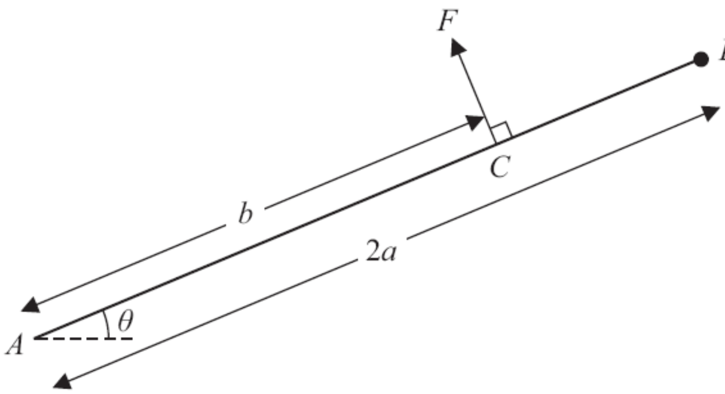


Figure 3

A uniform rod *AB*, of mass *m* and length $2a$, is freely hinged to a fixed point *A*. A particle of mass *m* is attached to the rod at *B*. The rod is held in equilibrium at an angle θ to the horizontal by a force of magnitude *F* acting at the point *C* on the rod, where $AC = b$, as shown in Figure 3. The force at *C* acts at right angles to *AB* and in the vertical plane containing *AB*.

(*a*) Show that $F = \dfrac{3amg\cos\theta}{b}$.

(4)

(*b*) Find, in terms of *a*, *b*, *g*, *m* and θ,

(i) the horizontal component of the force acting on the rod at *A*,

(ii) the vertical component of the force acting on the rod at *A*.

(5)

Given that the force acting on the rod at *A* acts along the rod,

(*c*) find the value of $\dfrac{a}{b}$.

(4)

6.

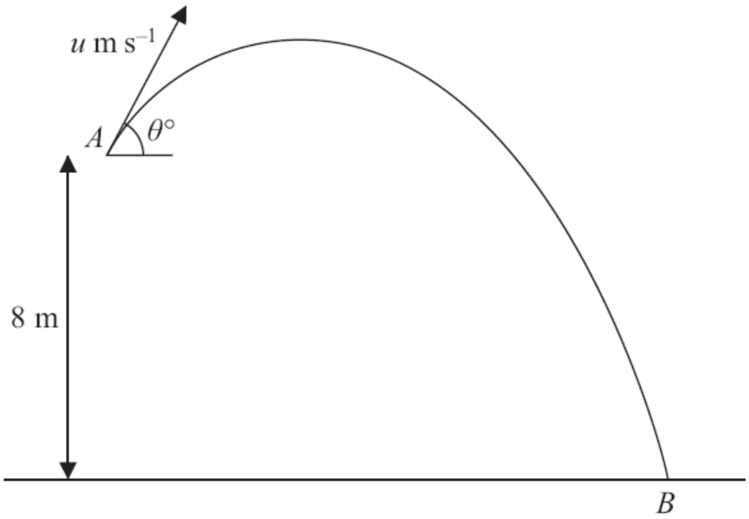


Figure 4

A ball is projected from a point A which is 8 m above horizontal ground as shown in Figure 4. The ball is projected with speed u m s^{-1} at an angle $\theta°$ above the horizontal. The ball moves freely under gravity and hits the ground at the point B. The speed of the ball immediately before it hits the ground is $2u$ m s^{-1}.

(*a*) By considering energy, find the value of u.

(5)

The time taken for the ball to move from A to B is 2 seconds. Find

(*b*) the value of θ,

(4)

(*c*) the minimum speed of the ball on its path from A to B.

(2)

7. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q and R have masses $2m$, $3m$ and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(*a*) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1 + e)u$.

(6)

After the collision between P and Q there is a direct collision between Q and R.

Given that $e = \frac{3}{4}$, find

(*b*) (i) the speed of Q after this collision,

(ii) the speed of R after this collision.

(6)

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(*c*) Find V in terms of u.

(3)

TOTAL FOR PAPER: 75 MARKS

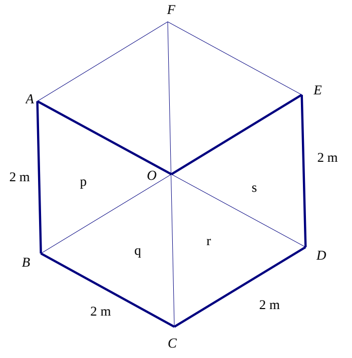
END

Question Number	Scheme	Marks	Notes
1.	Use of **I** = m**v**-m**u**	M1	Must be subtracting. Condone subtraction in the wrong order
	2**v** = (3**i** + 6**j**) +2(**i** – 4**j**)	A1	Correct unsimplified equation (= 5**i** – 2**j)**
	v = 2.5**i** - **j**	A1	
	Speed = $\sqrt{2.5^2+1^2}=\sqrt{7.25}(=2.69\,(\text{m s}^{-1}))$	M1	Use of correct Pythagoras with their **v**
		A1	Exact form or 2s.f. or better. Watch out for fortuitous answers from 2.5**i** + **j.**
		[5]	

Question Number	Scheme	Marks	Notes
2a	Work done = $15\mu R = 15\times 0.4\times 3g\cos 20°$ $= 18g\cos 20 = 166$ (J)	M1 M1 A1 [3]	$F_{max} = \mu\times 3g\cos 20$ (11.05). R must be resolved but condone trig confusion. $15\times$ their F_{max}. Independent M $15\times\ F_{max} + ...$ is M0 or 170 (J)
2b	Energy: WD against F + GPE + final KE = initial KE their WD $+ 3g\sin 20°\times 15 + \frac{1}{2}3v^2 = \frac{1}{2}3\times 20^2$ $v = 13.7$ (m s $^{-1}$)	M1A2ft A1 [4]	Must include all four correct terms (including resolving). Condone sign errors and trig confusion. Any sign errors in the KE terms count as a single error. Follow their WD -1ee Follow their WD or 14
Or 2b	$3a = -\ 0.4\times 3g\cos 20 + 3g\sin 20$ and use of $v^2 = u^2 + 2as$ $v^2 = 20^2 + 2\times a\times 15 (= 188.93...)$ $v = 13.7$ (m s $^{-1}$)	M1 A1ft A1ft A1 [4]	Complete method. Their F_{max} +component of weight A correct equation with their F_{max}. Allow for $a = +7.03...$ acting down the slope $a = -7.035....$ Correct equation for their a or 14 (m s $^{-1}$)

Question Number	Scheme	Marks	Notes
3a	$v = 0 = 2t^2 - 14t + 20$ $= 2\ t-2\ \ t-5$ $t = 2$ or $t = 5$	M1 M1 A1 [3]	Set $v = 0$ Solve for t
	There are many different approaches to part (b). The allocation of the two M marks is M1: A method to find the time when the velocity is a minimum M1: Evaluate the speed at that time		
e.g. b	$t = 0$, $v = 20$ (m s^{-1}) $a = 4t - 14 = 0$ $t = \frac{7}{2}$, $v = 2 \times \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1 [5]	 Must see ± 4.5 Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
balt1	$t = 0$, $v = 20$ (m s^{-1}) Sketch with symmetry about their $t = 3.5$ v(their 3.5) -4.5 Max speed = 20 ms^{-1}	B1 M1 M1 A1 A1 [5]	 Evaluate v at min. Correct work Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
b alt 2	$t = 0$, $v = 20$ (m s^{-1}) Justification of minimum or tabulate sufficient values to confirm location Evaluate v at min. Correct work Correct conclusion. Depends on the two M marks	B1 M1 M1 A1 A1 [5]	 Clearly stated & from correct solution only.

Question Number	Scheme	Marks	Notes
b alt 3	$t=0$, $v=20$ (m s^{-1})	B1	
	Complete the square as far as $\left(t-\frac{7}{2}\right)^2$	M1	
	$2\left(t-\frac{7}{2}\right)^2-\frac{9}{2}$	M1A1	
	Max speed = 20 ms^{-1}	A1 [5]	Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
c	$\int 2t^2-14t+20\,\mathrm{d}t=\frac{2}{3}t^3-7t^2+20t(+C)$	M1	Integration. Need to see majority of powers going up
		A1	All correct. Condone C missing
	Distance = $\left[\frac{2}{3}t^3-7t^2+20t\right]_0^2-\left[\frac{2}{3}t^3-7t^2+20t\right]_2^4$	M1	Correct method to find the distance, for their 2
		A1	Correct unsimplified
	$=2\times\left[\frac{2}{3}t^3-7t^2+20t\right]^2-\left[\frac{2}{3}t^3-7t^2+20t\right]_4$		
	$=2\left[\frac{16}{3}-7\times4+40\right]-\left[\frac{2\times64}{3}-7\times16+80\right]=24$ (m)	A1 [5]	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th><th>Notes</th></tr>
<tr><td>4a</td><td>

<table>
<tr><td>AOCB</td><td>OCDE</td><td>whole</td></tr>
<tr><td>1</td><td>1</td><td>2</td></tr>
<tr><td>1/2</td><td>1/2</td><td>$\bar{y}$</td></tr>
</table>
$2\bar{y} = 1 \times \frac{1}{2} + 1 \times \frac{1}{2}$
$\bar{y} = 0.5$ (m)</td><td>B1
B1

M1
A1

A1
[5]</td><td>For a valid division into basic elements: e.g. pair of rhombuses

Correct mass ratios for parts and the arrow shape
Correct vertical distances from a horizontal axis

Moments equation about a horizontal axis
Correct equation for their axis</td></tr>
<tr><td>a alt 2</td><td><table>
<tr><td>AOB</td><td>OBCD</td><td>DOE</td><td>whole</td></tr>
<tr><td>1</td><td>2</td><td>1</td><td>4</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>$\bar{y}$</td></tr>
</table>
$4\bar{y} = 2 \times 1$
$\bar{y} = 0.5$ (m)</td><td>B1
B1

M1A1

A1
[5]</td><td>Rhombus + two triangles

Moments equation</td></tr>
</table>

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th><th>Notes</th></tr>
<tr><td>a alt 3</td><td>
<table>
<tr><td>Hexagon</td><td>AOEF</td><td>whole</td></tr>
<tr><td>6</td><td>2</td><td>4</td></tr>
<tr><td>0</td><td>-1</td><td>$\bar{y}$</td></tr>
</table>
$4\bar{y} = 0 - \; -2 \times 1$

$\bar{y} = 0.5$ (m)
</td><td>B1
B1

M1A1

A1
[5]</td><td>Hexagon – rhombus</td></tr>
<tr><td>a alt 4</td><td>
h = height of each triangle = $\sqrt{3}$

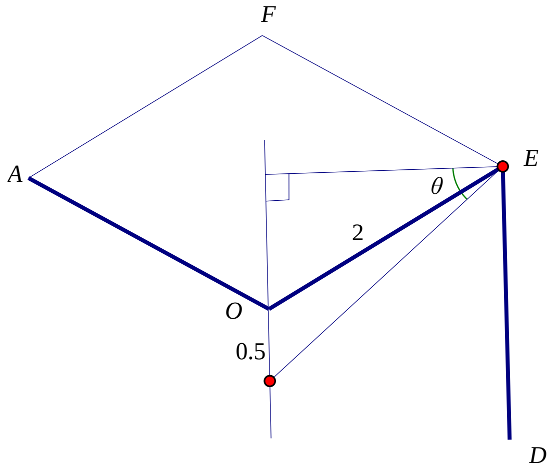
Distances of c of m from horizontal through O

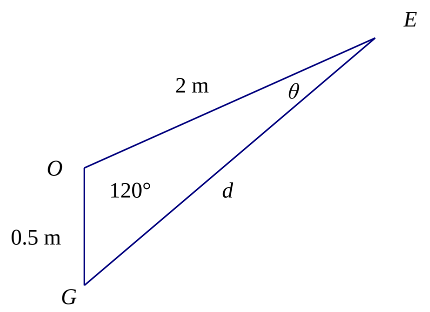
<table>
<tr><td>p</td><td>q</td><td>r</td><td>s</td><td>whole</td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>4</td></tr>
<tr><td>0</td><td>$\frac{2}{3}h\cos 30$</td><td>$\frac{2}{3}h\cos 30$</td><td>0</td><td>$\bar{y}$</td></tr>
</table>
$4\bar{y} = 2 \times 1 \times \frac{2\sqrt{3}}{3}\cos 30 \left(= \frac{4\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = 2 \right)$

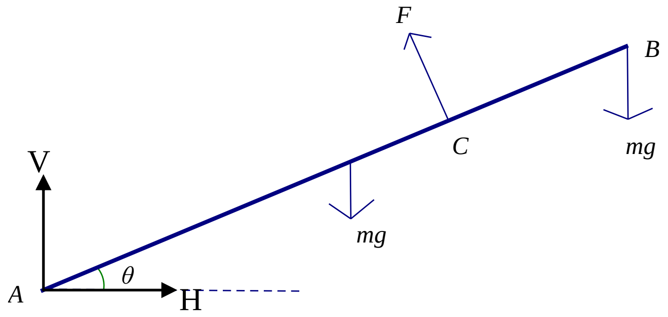
$\bar{y} = 0.5$ (m)
</td><td>B1
B1

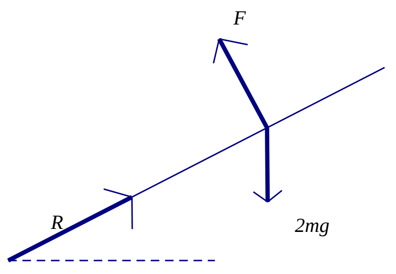
M1A1

A1
[5]</td><td>4 triangles</td></tr>
</table>

Question Number	Scheme	Marks	Notes
4b	In 4(b) the first two marks are M1: Indentify a triangle, with one angle correct, and attempt to find the lengths of two sides A1ft: 2 sides correct, follow their answer to (a) DM1: Work sufficient to be able to go on to find the required angle. Dependent on the preceding M1 A1ft: follow their answer to (a) DM1: Find the required angle. Dependent on the preceding M1 A1 Correct answer …. for example ……		
			
	$2\cos 30 = \sqrt{3}$, $"0.5" + 2\sin 30 = 1.5$	M1A1ft	Their 0.5 & their √3
	$\tan\theta = \frac{\text{their } 1.5}{\text{their } \sqrt{3}}$	DM1 A1ft	Use of tan in a right angled triangle. Accept the reciprocal Correct for their angle. Ft their 0.5
	Required angle = $\theta - 30 = \tan^{-1}\frac{1.5}{\sqrt{3}} - 30 = 40.89... - 30 = 10.9°$	DM1 A1 [6]	Correct strategy to find required angle e.g. $"\theta" - 30°$ or $90° - 30° - "\theta"$ Accept 11° , 10.9° or better

Question Number	Scheme	Marks	Notes
4balt			
	SAS in a relevant triangle	M1A1ft	Their 0.5
	$d^2 = 2^2 + 0.5^2 - 2\times 2\times 0.5\cos 120 = 5.25$	DM1 A1ft	Correct cosine rule. Correct equation. Their 0.5
	$\dfrac{\sin\theta}{0.5} = \dfrac{\sin 120}{\sqrt{5.25}}$	DM1	
	$\theta = 10.9°$	A1 [6]	

Question Number	Scheme	Marks	Notes
5a			
	Moments about A:	M1	Moments about A. Requires all three terms and terms of correct structure (force x distance). Condone consistent trig confusion
	$bF = a\cos\theta mg + 2a\cos\theta mg (= 3a\cos\theta mg)$	A2	-1 each error
	$F = \dfrac{3amg\cos\theta}{b}$ ***Answer given***	A1 [4]	
5b	$\rightarrow$: $H = F\sin\theta = \dfrac{3amg\cos\theta\sin\theta}{b}$	M1 A1	Resolve horizontally. Condone trig confusion RHS correct. Or equivalent.
	$\uparrow$: $2mg = \pm V + F\cos\theta$	M1 A1	Resolve vertically. Condone sign error and trig confusion Correct equation
	$\pm V = 2mg - \dfrac{3amg\cos\theta}{b} \times \cos\theta \left(= 2mg - \dfrac{3amg\cos^2\theta}{b} \right)$	A1 [5]	RHS correct. Or equivalent

Question Number	Scheme	Marks	Notes
5c	$\dfrac{2mg - \dfrac{3amg\cos^2\theta}{b}}{\dfrac{3amg\cos\theta\sin\theta}{b}} = \tan\theta$ $\dfrac{2b - 3a\cos^2\theta}{3a\cos\theta\sin\theta} = \dfrac{\sin\theta}{\cos\theta}$ $\Rightarrow 2b - 3a\cos^2\theta = 3a\sin^2\theta \Rightarrow 2b = 3a\ ,\ \dfrac{a}{b} = \dfrac{2}{3}$	M1 A1 DM1 A1 [4]	Use of tan, either way up. *V*, *H*, *F* substituted. Correct for their components in θ only Simplify to obtain the ratio of a and b, or equivalent
5c alt 2	The centre of mass of the combined rod + particle is $\dfrac{3}{2}a$ from *A*  3 forces in equilibrium must be concurrent $\Rightarrow b = \dfrac{3}{2}a$ $\Rightarrow \dfrac{a}{b} = \dfrac{2}{3}$	M1A1 M1 A1 [4]	Not on the spec, but you might see it.
alt c 3	*R* acts along the rod, so resolve forces perpendicular to the rod. $F = mg\cos\theta + mg\cos\theta$ $2mg\cos\theta = \dfrac{3amg\cos\theta}{b}$ $\Rightarrow \dfrac{a}{b} = \dfrac{2}{3}$	M1 A1 DM1 A1 [4]	Resolve and substitute for *F* Eliminate θ

Question Number	Scheme	Marks	Notes
alt c 4	R acts along the rod. Take moments about C $mg\cos\theta(2a-b) = mg\cos\theta(b-a)$ $2a-b=b-a$, $\Rightarrow \frac{a}{b}=\frac{2}{3}$	M1 A1 DM1A1 [4]	Moments about B gives $(2a-b)F = amg\cos\theta$ and substitute for F
c alt 5	Resultant parallel to the rod $\Rightarrow R = 2mg\sin\theta$ And $V^2+H^2=R^2$ $(2mg\sin\theta)^2 = \left(\frac{3amg\cos\theta\sin\theta}{b}\right)^2 + \left(2mg - \frac{3amg\cos^2\theta}{b}\right)^2$ Eliminate θ $\Rightarrow \frac{a}{b}=\frac{2}{3}$	M1 A1 DM1 A1 [4]	Substitute for V, H and R in terms of θ

Question Number	Scheme	Marks	Notes
6a	Conservation of energy: $\frac{1}{2}mu^2 + mg \times 8 = \frac{1}{2}m\ 2u\ ^2$ $mu^2 + 16mg = 4mu^2$ $16mg = 3mu^2, \quad u = \sqrt{\frac{16g}{3}}$ $u = 7.2$	M1 A2 -1ee DM1 A1 [5]	Energy equation must contain the correct terms, but condone sign error. Correct unsimplified Solve for u Accept 7.23. Accept $\sqrt{\frac{16g}{3}}$
6b	Vertical distance: $-8 = u\sin\theta \times 2 - \frac{g}{2} \times 4$ $\sin\theta = \frac{2g-8}{2u} = 0.802...$ $\theta = 53.3°$	M1 A2 -1ee A1 [4]	Condone sign errors or trig error. u must be resolved. Correct equation for their u. or 53°
6c	Min speed at max height, i.e. $u\cos\theta$ = 4.3 (m s^{-1})	M1 A1 [2]	Condone consistent trig confusion with part (b) or 4.32 (ms^{-1})

Question Number	Scheme	Marks	Notes
7a	CLM: $2mu = 2mv + 3mw$ Impact: $w - v = eu$ Subst $v = w - eu$: $2u = 2(w - eu) + 3w = 5w - 2eu$ $w = \frac{2}{5}(1 + e)u$ ***Answer Given***	M1 A1 M1 A1 DM1 A1 (6)	All three terms required, but condone sign errors Condone sign error, but must be subtracting and *e* must be used correctly. Penalise inconsistent signs here. Solve for *w*. Requires the two preceding M marks
7b	$w = \frac{7u}{10}$ CLM: $3mw = 3mx + 4my$ and Impact: $y - x = \frac{3w}{4}$ Subst: $3w = 3x + 4\left(x + \frac{3}{4}w\right)$ $x = 0$, $y = \frac{3}{4}w = \frac{21}{40}u$	B1 M1A1 DM1 A1 A1 (6)	Seen, or implied by correct speeds. Both needed Solve for *x* or *y*. Dependent on the preceding M mark $0.525u$,
7c	$v = -\frac{u}{20}$ Speed of separation = $\frac{u}{20} + \frac{21u}{40} = \frac{23u}{40}$	B1 M1 A1 (3) [15]	Correct velocity of *P* Correct use of their values and substitute for *e*. Check directions carefully $0.575u$